64. Constructing an Open Box An open box with a square base is required to have a volume of 10 cubic feet.

(a) Express the amount A of material used to make such a box as a function of the length x of a side of the square base.

(b) How much material is required for a base 1 foot by 1 foot?

(c) How much material is required for a base 2 feet by 2 feet?

(d) Graph $A = A(x)$. For what value of x is A smallest?

Similar problem situation to #63, which I'm assigning for homework. But for #64, they're giving you slightly different info and are looking for slightly different things...

64. a. Let A = amount of material,
x = length of the base, h = height, and
V = volume.

$$V = x^2h = 10 \Rightarrow h = \frac{10}{x^2}$$

$$\text{Total Area } A = (\text{Area}_{\text{base}}) + (4)(\text{Area}_{\text{side}})$$

$$= x^2 + 4xh$$

$$= x^2 + 4x\left(\frac{10}{x^2}\right)$$

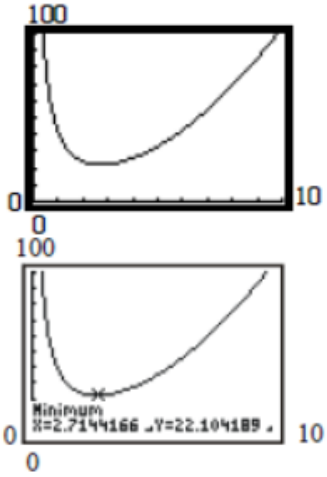
$$= x^2 + \frac{40}{x}$$

$$A(x) = x^2 + \frac{40}{x}$$

b. $A(1) = 1^2 + \frac{40}{1} = 1 + 40 = 41 \text{ ft}^2$

c. $A(2) = 2^2 + \frac{40}{2} = 4 + 20 = 24 \text{ ft}^2$

d. $y_1 = x^2 + \frac{40}{x}$



The amount of material is least when $x = 2.71$ ft.

1.4 Library of Functions; Piecewise-defined Functions

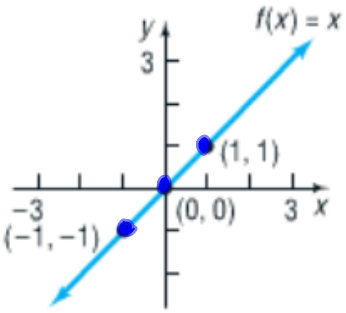
PREPARING FOR THIS SECTION

- Intercepts (Foundations, Section F.2, pp. 10–11)
- Graphs of Key Equations (Foundations, Section F.2: Example 3, p. 9; Example 10, p. 13; Example 11, p14; Example 12, p. 14)

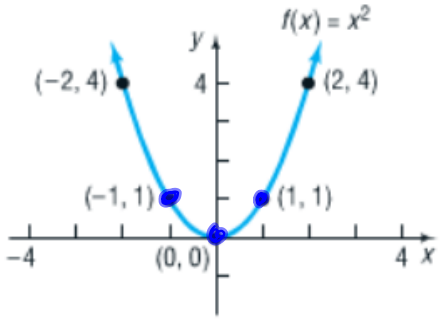
Objectives:

1 Graph the Functions Listed in the Library of Functions
2 Graph Piecewise-defined Functions

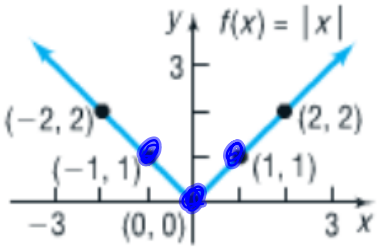
Identity Function



Square Function



Absolute Value Function

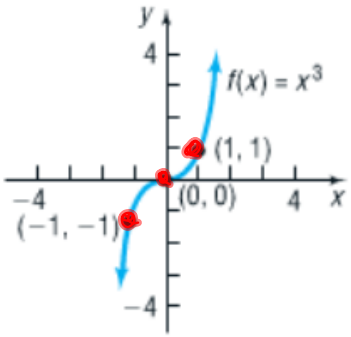


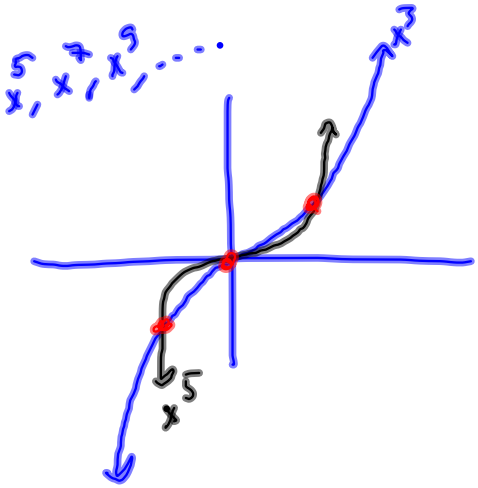
x	y = f(x) = |x|	(x, y)
0	0	(0, 0)
1	1	(1, 1)
2	2	(2, 2)
3	3	(3, 3)

Properties of $f(x) = |x|$

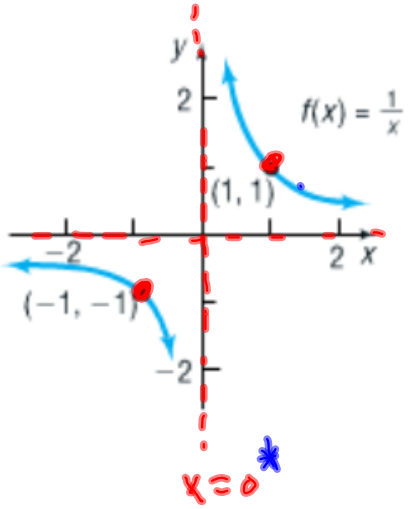
1. The x-intercept of the graph of $f(x) = |x|$ is 0. The y-intercept of the graph of $f(x) = |x|$ is also 0.
2. The function is even.
3. It is decreasing on the interval $(-\infty, 0)$.
   It is increasing on the interval $(0, \infty)$.
4. It has a local minimum of 0 at $x = 0$.

Cube Function





Reciprocal Function



$f(-x) = \frac{1}{(-x)} = -\frac{1}{x} = -f(x)$

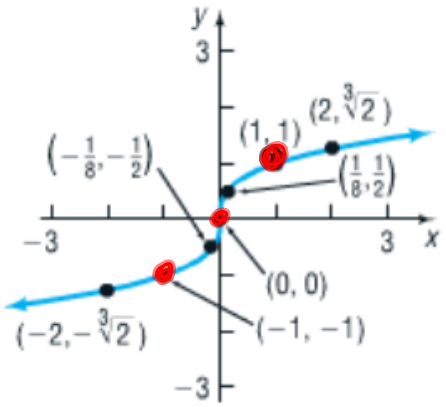
$y = 0$

$x = 0$

$\frac{1}{100000}$

$\sqrt[3]{x} = x^{\frac{1}{3}}$

Figure 27

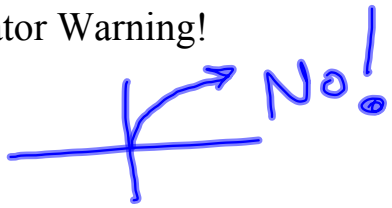


x	$y = f(x) = \sqrt[3]{x}$	(x, y)
0	0	(0, 0)
$\frac{1}{8}$	$\frac{1}{2}$	$\left(\frac{1}{8}, \frac{1}{2}\right)$
1	1	(1, 1)
2	$\sqrt[3]{2} \approx 1.26$	$\left(2, \sqrt[3]{2}\right)$
8	2	(8, 2)

Properties of $f(x) = \sqrt[3]{x}$

1. The x-intercept of the graph of $f(x) = \sqrt[3]{x}$ is 0. The y-intercept of the graph of $f(x) = \sqrt[3]{x}$ is also 0.
2. The function is odd.
3. It is increasing on the interval $(-\infty, \infty)$.
4. It does not have a local minimum or a local maximum.

Graphing Calculator Warning!



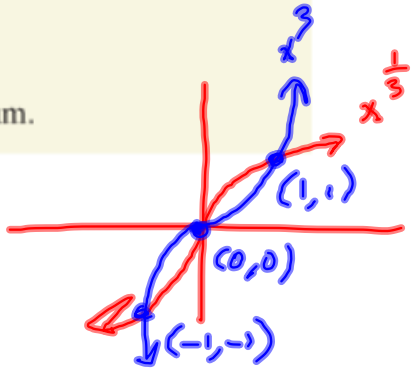
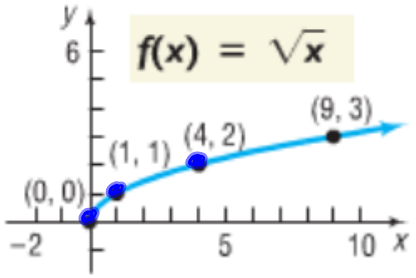


Figure 26



Properties of $f(x) = \sqrt{x}$

1. The x-intercept of the graph of $f(x) = \sqrt{x}$ is 0. The y-intercept of the graph of $f(x) = \sqrt{x}$ is also 0.
2. The function is neither even nor odd.
3. It is increasing on the interval $(0, \infty)$.
4. It has a minimum value of 0 at $x = 0$.